Residual stress in a thick steel weld determined using the contour method

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Abstract:

The contour method was used to determine the residual stress field in a welded steel plate. Different techniques for developing the deformed surface were studied in an attempt to generate the most accurate results. The order of the surface fit was varied, as well as the method for extrapolating profile data near the plate edges. Care was taken to insure that two surface profiles from opposite sides of the cut were properly aligned prior to averaging. Surface fitting was done using MATLAB and the finite element analysis was conducted using ABAQUS. Example code and input files are included as appendices.

INTRODUCTION

This report summarizes an application of the contour method to measure the weld-direction component of residual stress in a 38 mm thick, multipass steel weld. Residual stresses in welds can significantly affect the mechanical performance of the structures in which they exist. Processes particularly impacted by residual stresses include corrosion, fatigue, and fracture. The manner in which residual stresses affect these failure processes is often difficult to ascertain because the residual stresses are difficult to measure. The contour method has recently been developed and has the capability to determine a two-dimensional map of the residual stress component normal to a plane through an object. This report discusses an application of the contour method to the measurement of the weld-direction residual stress a thick multipass steel weld.

The contour method relies on deformations that occur when a part containing residual stress is cut along a plane. Assuming that the cut path is planar, any variation of the cut faces from a plane is assumed to be the result of residual stress. In order to cut on a path which is as planar as possible and to remove as little material as possible, wire electric discharge machining

(WEDM) is used. During cutting, the part is held in place so that deformations are restrained as much as possible during cutting. Following cutting, the cut surfaces on each of the two halves of the part are measured in order to determine the surface profile normal to the cut. Averaging of the surface profiles measured on the two halves reduces error from both shear stress existing on the cut plane and from variations of the cut path from a plane. The average surface profile, once obtained, can then be used to determine the residual stress component normal to the cut path existing in the part prior to cutting. This step is performed with the aid of the finite element method (FEM). The average surface profile is used to determine nodal displacements applied normal to the cut face on a finite element model of the cut part. Stresses determined by this FEM analysis provide an estimate of the residual stress prior to cutting.

While the contour method is simple in concept, its reliance on imperfect processes for cutting and measurement introduces challenges in application. Surface profiles are typically measured with a coordinate measuring machine (CMM) or a laser range finder. These devices produce a discrete set of data points (i.e., coordinate triples (x,y,z)) each of which is subject to error due to precision and bias. In addition, the surface produced by WEDM, while smooth by some standards, can have significant roughness compared with the range of variation exhibited in the surface profile. Therefore, raw profile data are mathematically fit to a smooth surface in an effort to mitigate the effects of roughness and point-wise uncertainty.

We employed the contour method to determine the weld-direction residual stress (i.e., σ_{zz}) in the welded joint shown in Figure 1. The joint was cut normal to the weld direction using WEDM and the cut surface profiles were measured using a CMM. The main objective of this study was to determine residual stress in the weld using a smoothed average surface. In addition, the effects of several steps in data processing were investigated. Stresses were determined from profile data not fit to a smooth surface to ascertain the impact of smoothing. Stresses were determined from smooth fits to profile data from each surface of the cut separately to determine the effect of surface averaging. The effect of erroneous data near the edges of the surface profiles was also investigated by comparing stresses computed when the regions of erroneous data were treated differently.

METHODS

Specimen and cut geometry

In this project, the contour method was used to determine the residual stress field on a cross section perpendicular to a steel weld (Figure 1). For this particular experiment the cut was made with a Hansvedt Model DS-2 Traveling Wire EDM machine using a 0.25 mm diameter brass wire. During the cutting process, the weld plate was clamped to a 44.5 mm thick aluminum plate to prevent movement. The approximate dimensions of the cut surfaces were 38 mm by 215mm. The coordinate system used for the analysis has the x-direction aligned with the corners of the weld joint on the concave side (Figure 1).

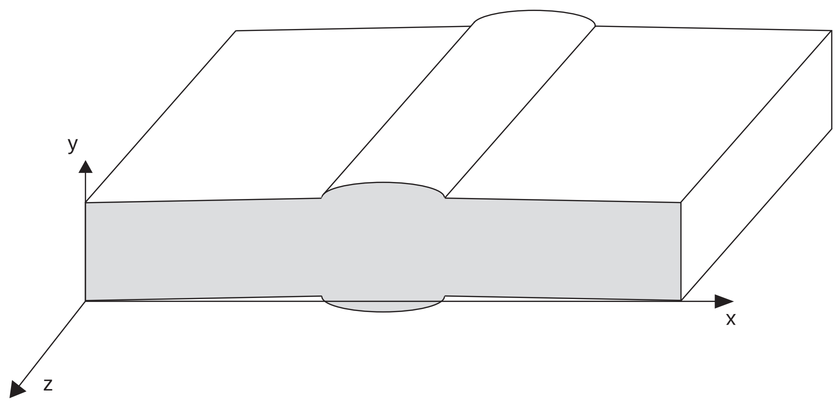


Figure 1 – Specimen and coordinate system

Surface profile measurement

Once the weld was cut in half, a CMM was used to measure the surface profile of both cut faces. Each profile was measured on a different CMM. One surface, identified here as “surface 1”, was measured with an International Metrology Systems Impact II CMM equipped with a 1-mm diameter ruby tip. The other surface, “surface 2”, was measured using a Brown & Sharpe XCEL 765 CMM equipped with a 1-mm diameter ruby tip. The path of the CMM probe on each surface varied considerably (Figure 2). Measurements were taken over nearly the entire surface of the cut for each half of the weld.

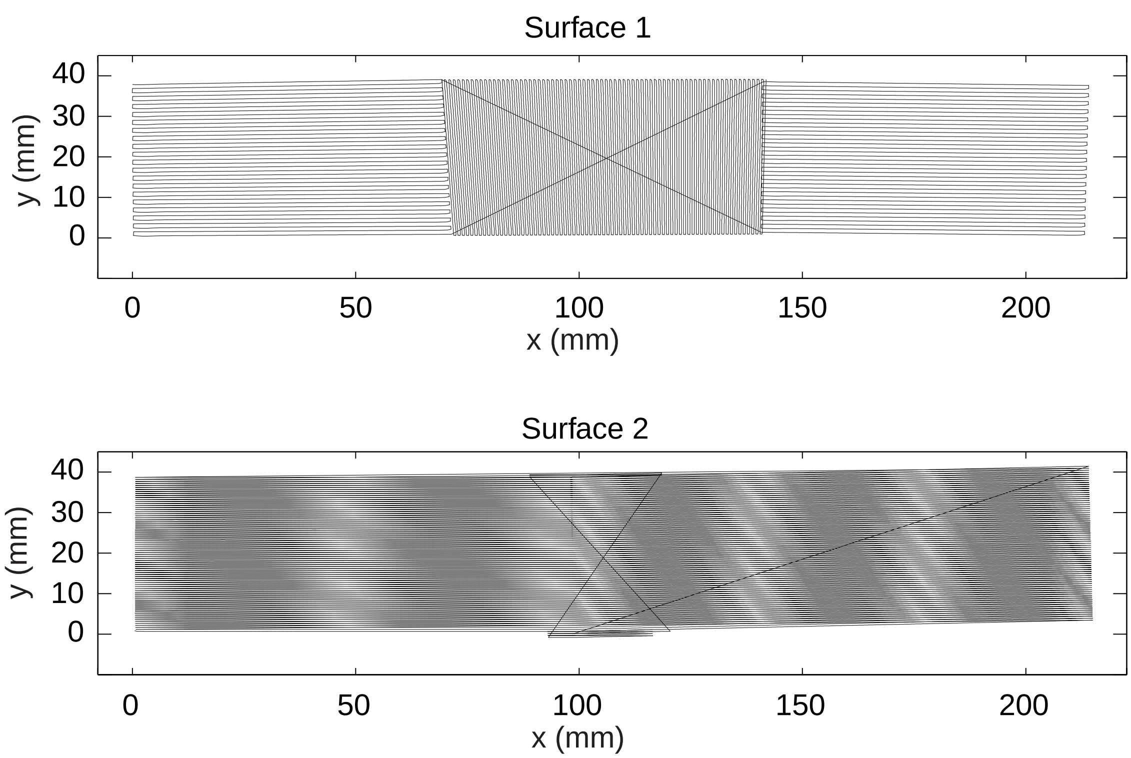


Figure 2 – Path of the CMM probe on surface 1 (top) and surface 2

Surface alignment and averaging

Because the CMM path varied considerably for the two cut surfaces, and because the curvature of the weld shown in Figure 1 suggests that surfaces 1 and 2 have different orientations, the two surfaces were carefully aligned with each other prior to further analysis. In order to gain a better understanding of the alignment of the two surfaces, the surface profile data were plotted along constant coordinate lines for both data sets. From these line plots, the translation and rotation of the two data sets, needed to match the profile data from one half to that on the other, was confirmed. Once the data sets were properly aligned, surface profile data were obtained at a set of grid points within the weld geometry using Delaunay triangulation. The gridded profile data from surfaces 1 and 2 (z_1 and z_2) were then used to define the average surface profile

$$\hat{z}(x,y) = \tfrac{1}{2}[z_1(x,y) + z_2(x,y)] \tag{1}$$

Surface fitting

The average surface data were fit using a tensor product of one-dimensional Fourier series. The planar (x,y) grid coordinates were transformed into Fourier domain coordinates (ξ,η) covering the range $[0,\pi]$ to allow for asymmetry

$$\begin{aligned} \xi &= \pi \frac{x - x_{min}}{x_{max} - x_{min}} \\ \eta &= \pi \frac{y - y_{min}}{y_{max} - y_{min}} \end{aligned} \tag{2}$$

The tensor product of n^{th} order Fourier series in (ξ,η) was given by

$$\begin{aligned} z(\xi,\eta) = a_0 &+ \sum_{k=1}^{n} [a_k \cos k\xi + b_k \cos k\eta + c_k \sin k\xi + d_k \sin k\eta] \\ &+ \sum_{k=1}^{n-1} [e_k \cos k\xi \cos(n-k)\eta + f_k \sin k\xi \cos(n-k)\eta \\ &+ g_k \cos k\xi \sin(n-k)\eta + h_k \sin k\xi \sin(n-k)\eta] \end{aligned} \tag{3}$$

where a_k, b_k, … , h_k were parameters of the fit, and where the summations were carried out only if the upper index was greater than or equal to the lower index. A surface fitting routine that determined the values of the fit parameters from the gridded surface profile was developed using MATLAB, a software package well suited to matrix manipulation. A copy of the MATLAB-language code is appended to this report. Surface profile data were fit over a range of orders from first to tenth. The total number of parameters in a fit of order n is $1+2n(n+1)$, so that the number of parameters in the fits ranged from 5 to 221.

Stress determination

A finite element mesh was constructed to determine residual stress from the surface profile. The mesh represented the weld geometry (Figure 3). For node points on the cut surface of the model, z-direction displacements were determined from the fit to the average surface profile. Unfortunately, CMM data near the edges of the cut surfaces were incomplete (Figure 4), and this complicated stress determination. Data were mainly lacking near the upper (i.e., y_{max}) and lower (i.e., y_{min}) edges of the weld. At nodes that fell outside the region of valid CMM data, a plateau function was used to extrapolate displacements from the region of valid data, where the displacement of nodes outside the region of valid data (Figure 4) was set equal to the displacement of the nearest node within the region of valid data. It was found that the plateau function had a minimum impact on the computed stress within the region of valid data when a planar component of the surface profile was first subtracted from the average surface profile fit. Therefore, prior to the finite element computation, the average surface profile fit was further fit to a plane

$$p(\xi,\eta) = p_0 + p_1\xi + p_2\eta + p_3\xi\eta \tag{4}$$

where p_i are the coefficients of the plane. Nodal displacements for the finite element analysis were then determined by subtracting the plane from the surface profile fit

$$\tilde{z}(\xi,\eta) = z(\xi,\eta) - p(\xi,\eta) \tag{5}$$

Since $p(\xi,\eta)$ represents a rigid body displacement, stresses determined from displacements $\tilde{z}(\xi,\eta)$ would be the same as displacements given by $z(\xi,\eta)$, except for the effect of the plateau used for missing data.

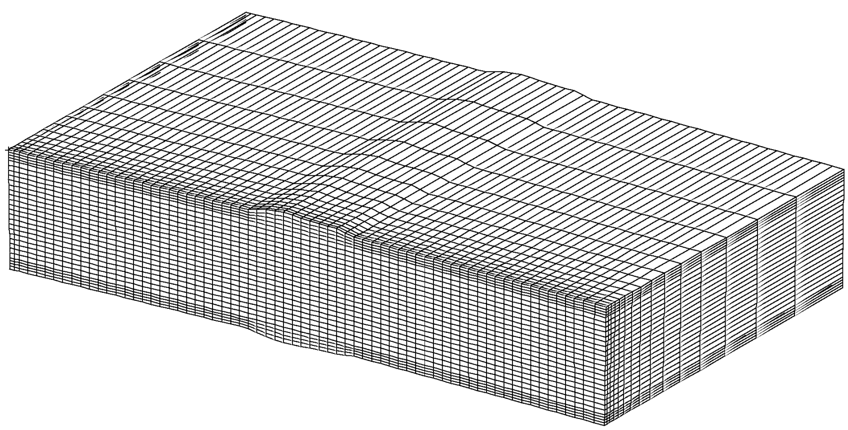


Figure 3 – Finite element mesh of the weld, displacements applied to face shown

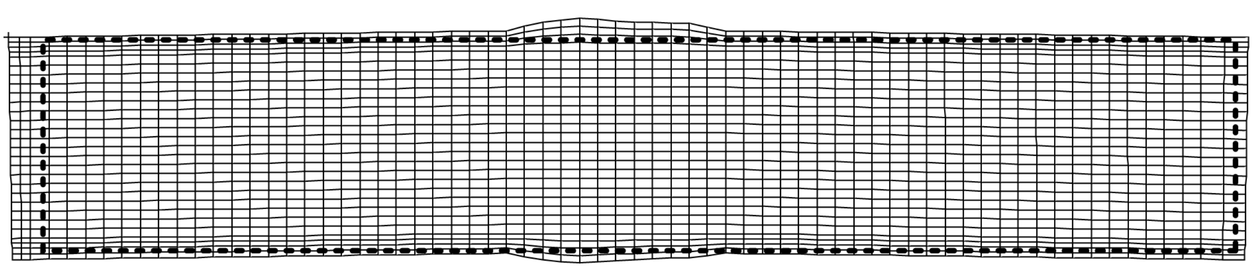


Figure 4 – Cut face in the FE mesh, CMM data were available inside the dashed rectangle

RESULTS

Surface profile measurement

The range of data from the CMM paths were approximately: $x \sim$ [-1 mm, 40 mm], $y \sim$ [0 mm, 215 mm], and $z \sim$ [-0.3 mm, 0.3 mm]. The CMM data for surface 1 consisted of roughly 17,000 points and surface 2 had roughly 33,000 points. On surface 2, the measurements were made about every 1 mm along the y-direction and about every 0.5 mm along the x-direction. The measurements on the center portion of surface 1 were taken with the same spacing as for surface 2 while the spacing in the area away from the center was approximately double the spacing used for surface 2 (Figure 2). The raw CMM data (Figure 5) exhibit several regions where the CMM probe apparently slipped off the edge of the surface. These regions show up as sharp peaks along the surface boundaries. Since the peaks do not represent the actual weld surface they were removed by truncating the surface datasets following surface alignment and prior to surface averaging and fitting.

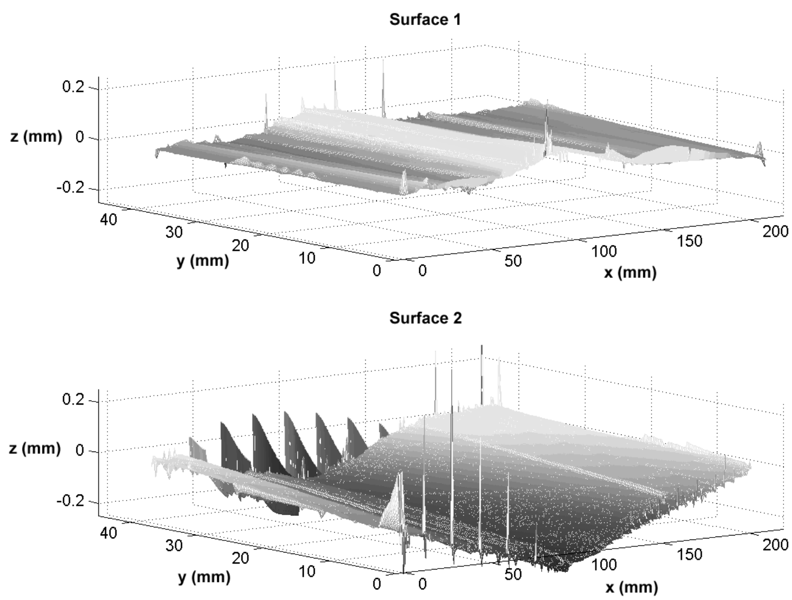


Figure 5 – CMM data for surface 1 (top) and surface 2

Surface alignment and averaging

Surface alignment was only needed for surface 2. When CMM data were taken on surface 1, the coordinate system used by the machine coincided with the coordinates shown in Figure 1. As suggested by the CMM probe path for surface 2 (Figure 2), the planar coordinates reported by the CMM (x_2,y_2) needed to be reflected, rotated, and translated to coincide with the coordinates for surface 1 (x_1,y_1) ≡ (x,y). The data for surface 2 were first reflected and translated according to

$$y_2' = 38.0mm - y_2 \tag{6}$$

The translation of 38.0 was determined from the y-coordinate of the upper left corner of surface 2, which should coincide with the coordinate origin. The negative sign on y_2 reflected the data about the x_2 axis, which was necessary since surface 2 is a mirror image of surface 1. The required rotation of surface 2 was determined from the lower left point and the lower right point of the (x_2, y_2') data, which should both lie on the x axis (Figure 1). The (x_2, y_2') data were rotated about the z_2 axis by 0.82° to match the coordinates assumed in the analysis

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} \cos(0.82°) & \sin(0.82°) \\ -\sin(0.82°) & \cos(0.82°) \end{bmatrix} \begin{Bmatrix} x_2 \\ y_2' \end{Bmatrix} \tag{7}$$

Figure 6 shows a top view of the data sets following surface alignment. Points where the CMM probe apparently fell off the surface, and therefore produced erroneous data, were excluded from the analysis, and the remaining regions of valid data are shown by the dashed rectangles in Figure 6. All data outside these rectangles was not used in further analysis. The valid data was found in the ranges $x \sim [5\text{ mm}, 213\text{ mm}]$ and $y \sim [2\text{ mm}, 36.5\text{ mm}]$.

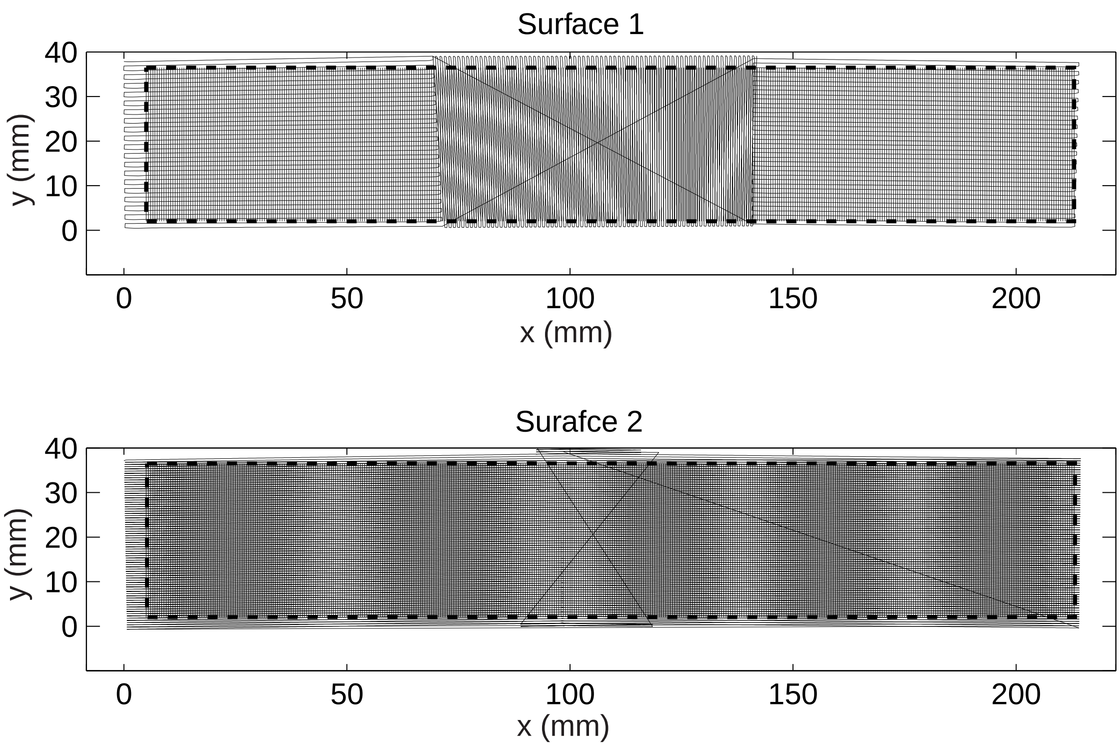


Figure 6 – CMM data on the two surfaces after aligning surface 2

Plots of the surface profiles after the completion of the surface alignment are shown in Figure 7 and Figure 8. At first glance these two surfaces do not appear like they are opposite sides of the same cut. The central peak on surface 1 is much steeper and narrower than the central peak on surface 2. Also, the height of the central peak on surface 2 is significantly higher than the height of the central peak on surface 1. However, after creating line plots for the surfaces it became apparent that they were in fact two halves of the same cut.

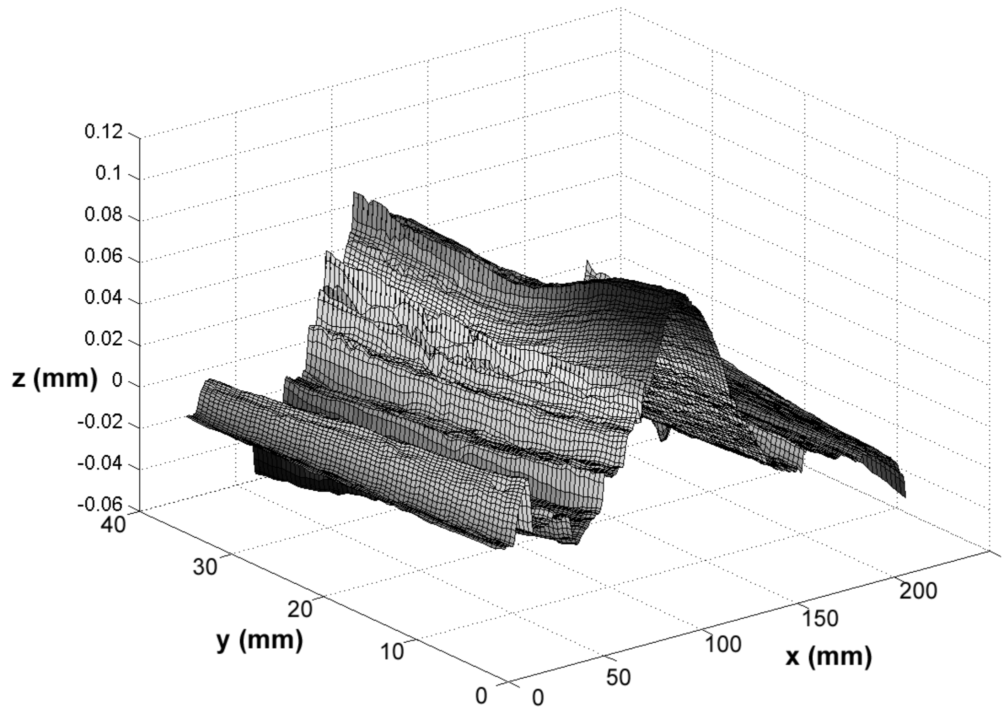


Figure 7 – Surface 1, after truncation of erroneous data

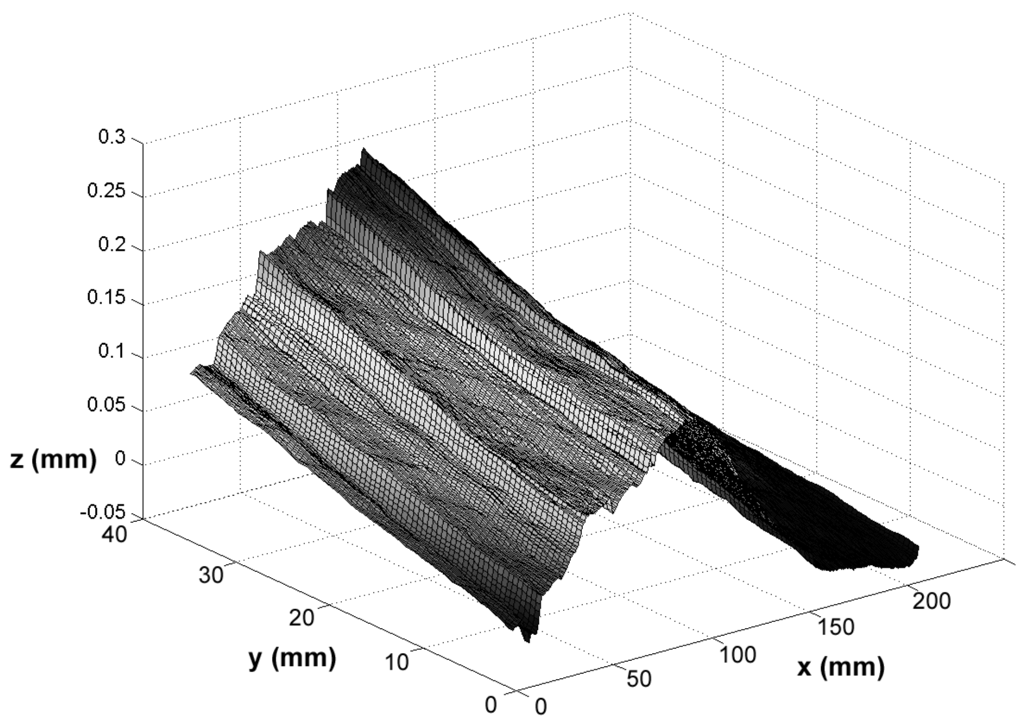


Figure 8 – Surface 2, after alignment and truncation of erroneous data

Line plots of the surface profiles were created to verify that the surfaces were properly aligned. Figure 9 shows line plots near each edge of the surface. Plots for constant *y* indicate good alignment because they exhibit mirror ridge features, which occur at the same values of *x*. However, plots for constant *x* are inconclusive because they lack any distinguishing features. Additional line plots for constant *y* are show in Figure 10, which also show that there is good surface alignment. The plot in the lower right-hand corner of Figure 10 was made with a very small range of *x* and the data for surface 2 was shifted downward to more closely illustrate the surface alignment. This plot indicated that the surfaces were aligned within 0.25 mm in the *x*-direction. Additional line plots for constant values of *x* were constructed in order to obtain similar confirmation of alignment in the *y* direction (Figure 11). As with the plots along constant values of *x* in Figure 9, the additional plots did not provide any conclusive indication of alignment.

Following alignment, the two surfaces were averaged. A grid was established covering the range of valid data with grid spacing of 0.5 mm in each direction. Delaunay triangulation was then employed to determine values of z_1 and z_2 at all grid points, and these values were averaged together to obtain the average surface profile (Figure 12). The degree of surface smoothing obtained by averaging was remarkable (compare Figure 12 with Figure 7 and Figure 8). The smoothing effect of the average is also shown in the line plots for constant values of *x* and *y* (Figure 9, Figure 10, and Figure 11).

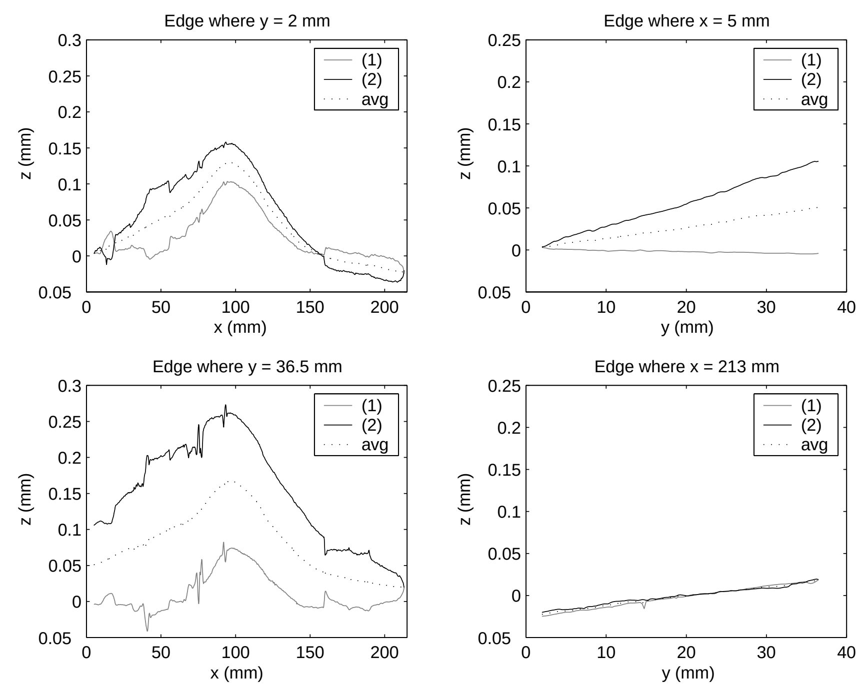


Figure 9 – Line plots near the edges of the surfaces

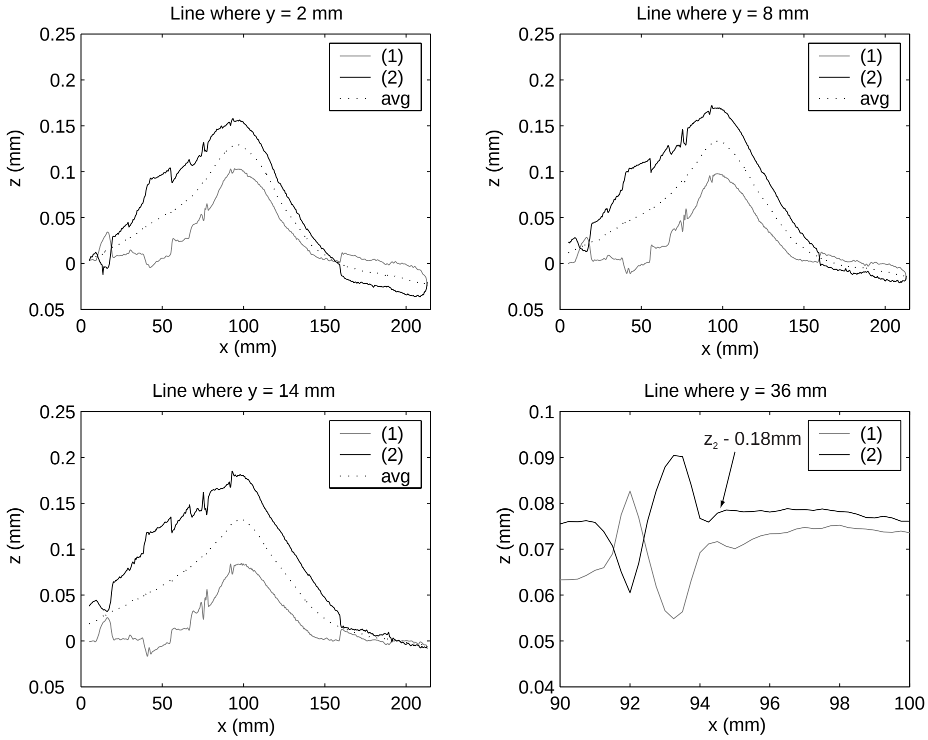


Figure 10 – Line plots for various values of y

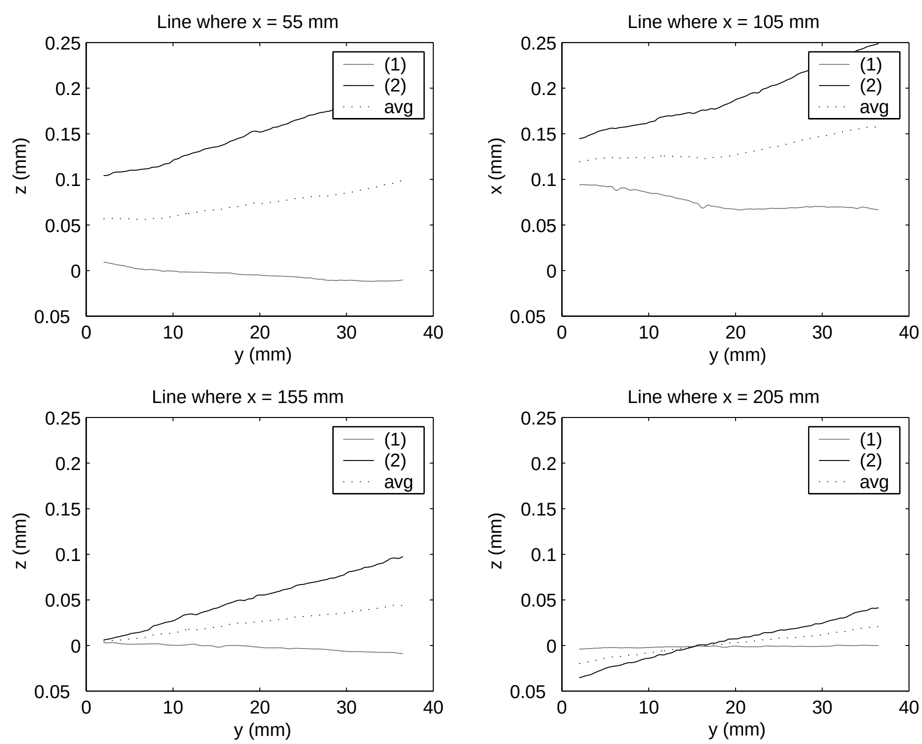


Figure 11 – Line plots for various values of x

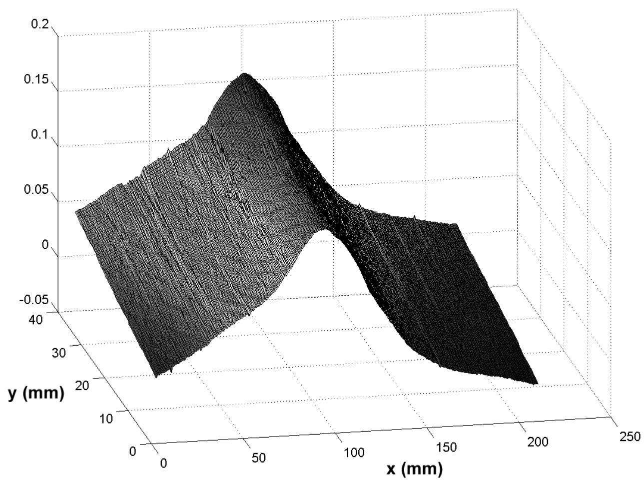


Figure 12 – Average of surface 1 and surface 2

Surface fitting

After the two surfaces had been averaged together they were fit to a Fourier surface using least squares. A convergence study was done to determine the order of Fourier surface required to adequately fit the data. The root mean square (RMS) error was plotted versus the order of fit assumed (Figure 13), which showed a plateau at 9^{th} order (181 terms). Line plots were created to illustrate the relationship between the order of the Fourier surface fit and the fit quality (Figure 14 and Figure 15). These plots reinforced the notion that a 9^{th} order fit yielded adequate fit quality. A surface plot (Figure 16) was also made to show the agreement between the average surface profile (shown in light) and the 9^{th} order Fourier surface fit (in dark).

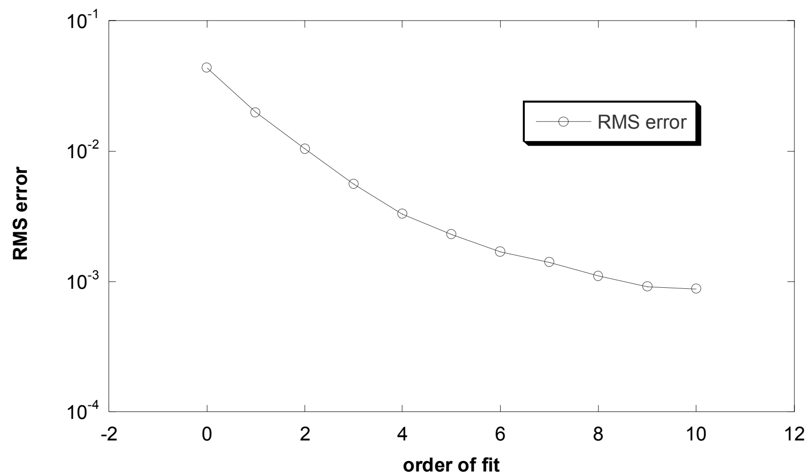


Figure 13 – RMS error between the fit and averaged surface data versus order of fit assumed

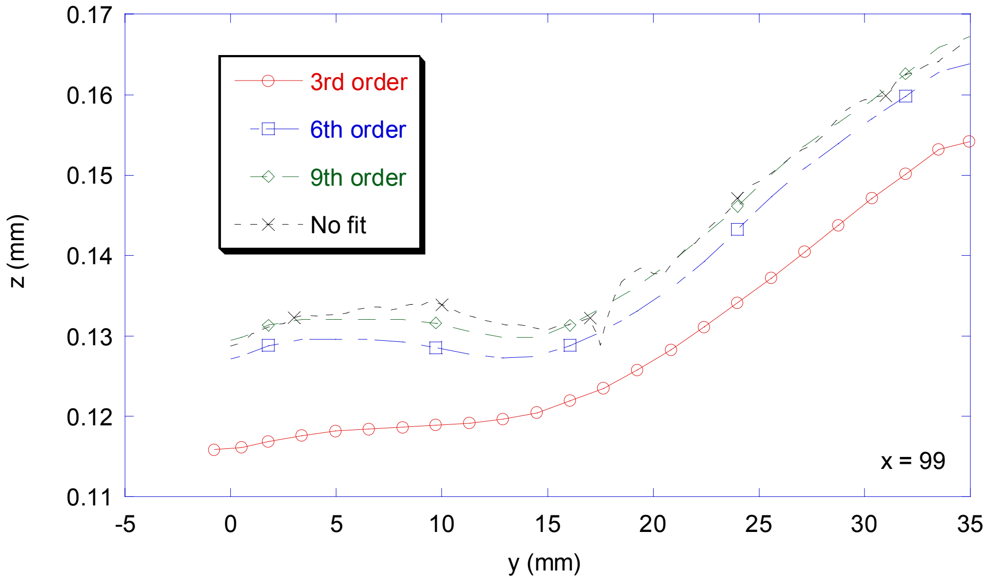


Figure 14 – Effect of order on the surface fit at x = 99 mm

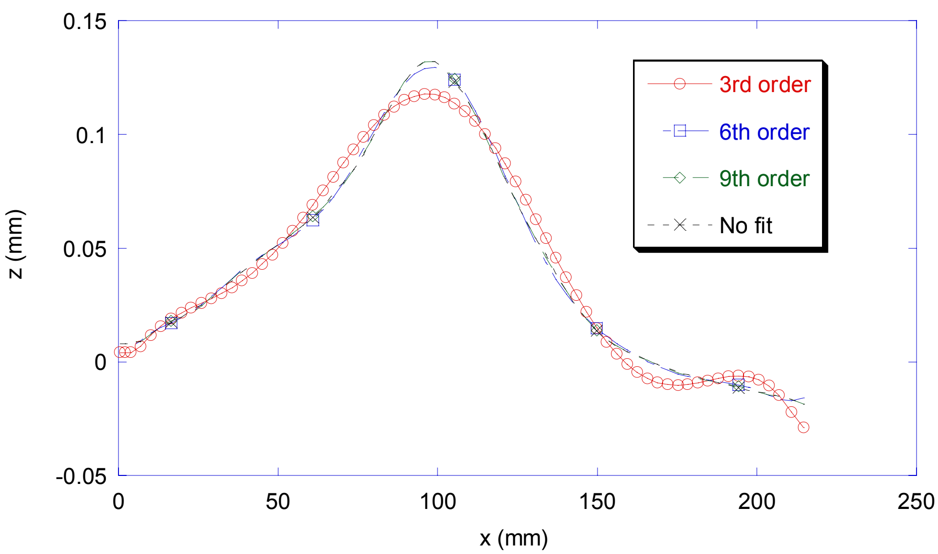


Figure 15 – Effect of order on the surface fit at y = 5.2 mm

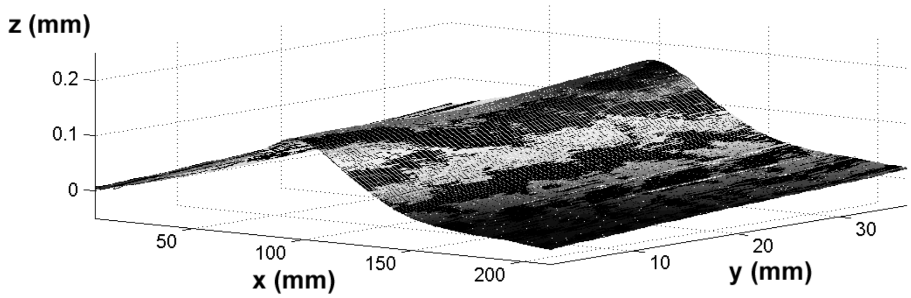


Figure 16 – 9th order Fourier surface and average surface

Stress determination

The Fourier surface was further fit to a plane in order to minimize errors induced by the plateau scheme used to extrapolate the data when performing the stress analysis. Figure 17 shows a plane that has been fit to the 9th order Fourier surface. Subtraction of the plane from the Fourier fit gives the surface shown in Figure 18. It is noteworthy that the *y*-direction slope of the surface fit has been reduced near the surface edges, where the plateau was necessary.

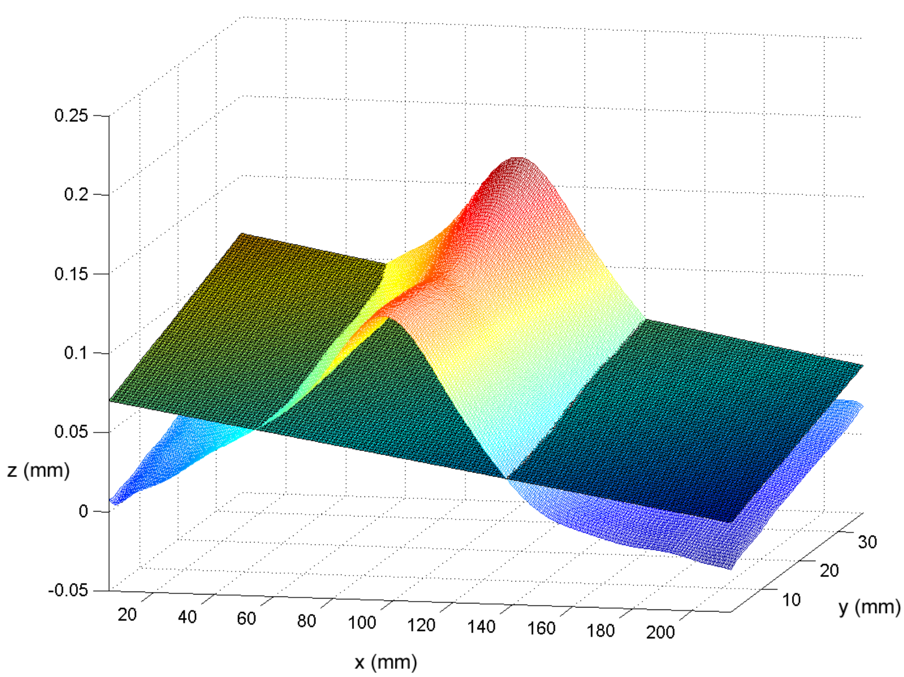


Figure 17– 9th order Fourier surface fit and planar component

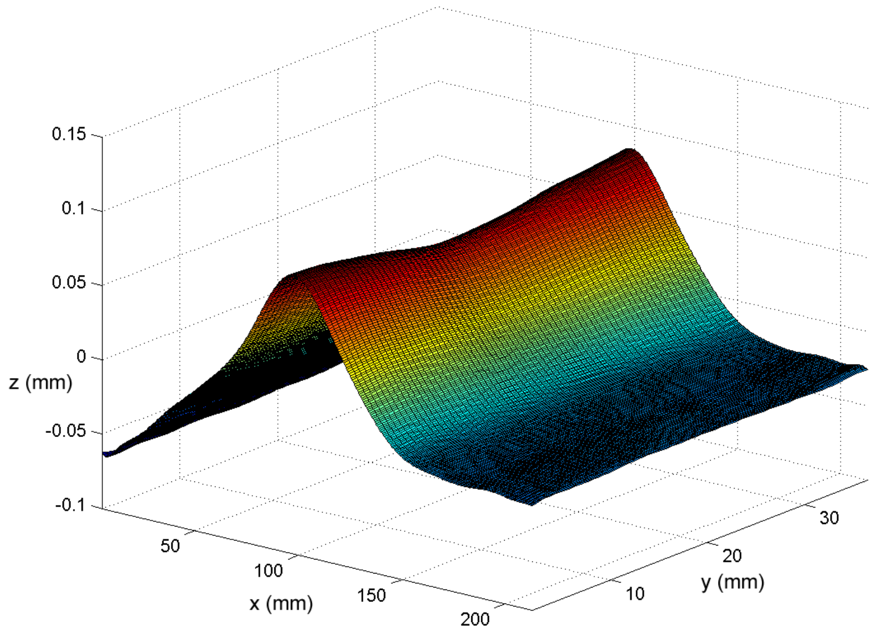


Figure 18 – 9th order Fourier surface fit after subtraction of planar component

Following surface fitting, the surface fit provided *z*-direction displacements for a finite element stress analysis. Displacements for nodes lying outside the region of valid surface data (outside the dotted line in Figure 4) were generated by using the plateau. The results of the stress analysis for the 9th order Fourier surface fit are shown in Figure 19. The residual stress field has an area of tension near the center of the weld where thermal effects would have been the greatest. The tensile stresses are balanced by a region of compressive residual stress outside the weld bead. The color scale for the residual stress contour plot is shown in Figure 20.

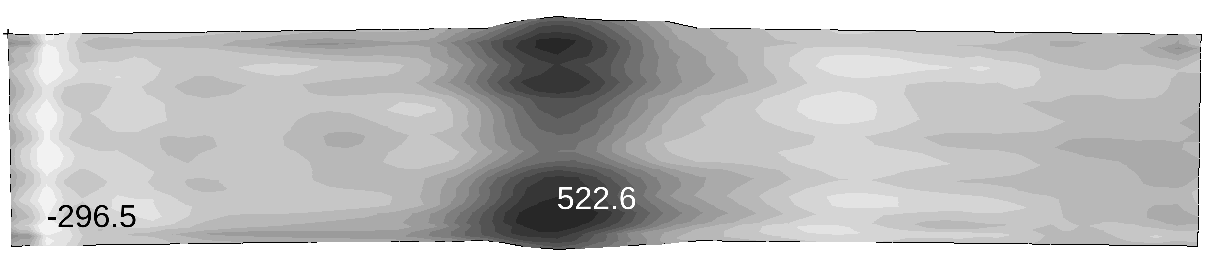


Figure 19 – Residual stress for a 9th order Fourier surface fit

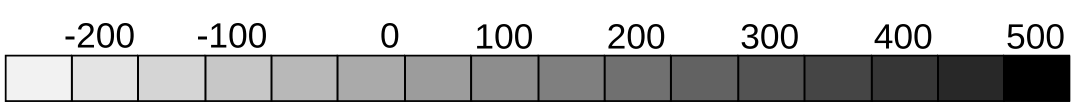


Figure 20 – Color map for residual stress contour plots, MPa (all plots use the same color map)

DISCUSSION

Surface alignment and averaging

Coordinate registration was time consuming because it was difficult to determine exactly how surface 2 had to be manipulated to match up with surface 1. However, once the data were carefully examined with attention to the asymmetry of the weld joint (Figure 1), alignment was fairly simple. The asymmetry of the weld was a benefit in this study, and application of the method to a perfectly symmetric geometry would be difficult if the surfaces and coordinates used were not carefully documented.

Ridges in the measured surface profiles, left by the cutting process, were helpful in verifying surface alignment. The cut was performed with the EDM wire running approximately along the y-direction, and the cut proceeded along the x direction. The path of the EDM wire was not straight, as evidenced by the mirror ridges in the two surfaces occurring at specific values of x (Figure 7 and Figure 8). The variation of the wire path produced excellent demarcations of the surface to assist in alignment along the x-direction. The lack of similar demarcations along the y-direction resulted in difficulty in aligning the surfaces in the y-direction.

Surface fitting

Surface fitting was the most difficult step of the analysis. One of the most challenging aspects was handling erroneous and missing data points near the surface edges. Figure 4 shows the regions where data were unknown. In this study, we truncated the erroneous data and later replaced it using a plateau of the surface fit. However, the residual stress field would ideally be obtained from surface profile data taken over the entire cut face. The effect of the treatment of erroneous data near the edges on the residual stress is discussed below.

Stress determination

Although the determination of residual stress is rather simple, the various assumptions and techniques used in obtaining the surface fit have an influence on the residual stress determined. Here we consider the effects of the order assumed for the surface fit, the out of plane length of the finite element model, and the method of extrapolation of the surface fit to nodes outside the region of valid data. The effects of surface fitting and surface averaging are also briefly described.

For comparison, the stress analysis was repeated for 6th order and 3rd order Fourier surface fits. The residual stress for the 6th order Fourier surface is shown in Figure 21 and the 3rd order results are shown in Figure 22. The 6th order results look similar to the 9th order results, while the 3rd order results are noticeably different from the 9th order results. Line plots of the residual stress along the line y = 5.2 mm (Figure 23) and along the line x = 99 mm (Figure 24) show how the order of the Fourier surface affects the calculated residual stress. The 9th order profile produced a more pronounced peak of residual stress than did the 6th order surface, but otherwise the stress distributions are in agreement. The 3rd order surface produces stresses that vary markedly from the other two results.

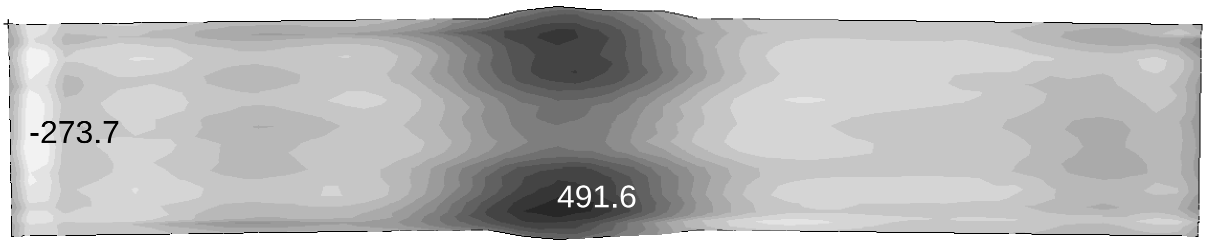


Figure 21 – Residual stress for 6th order Fourier surface

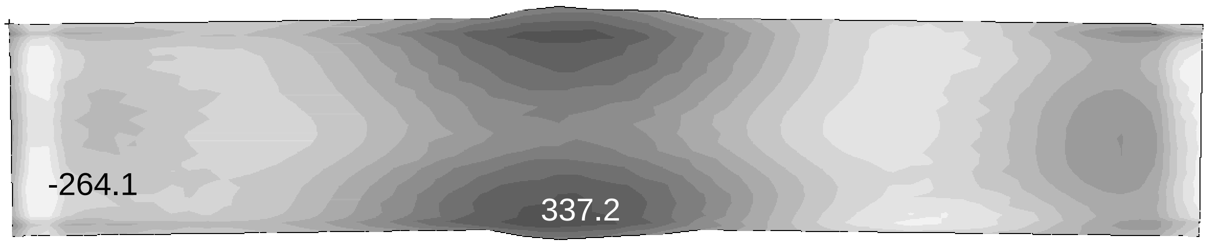


Figure 22 – Residual stress for 3rd order Fourier surface

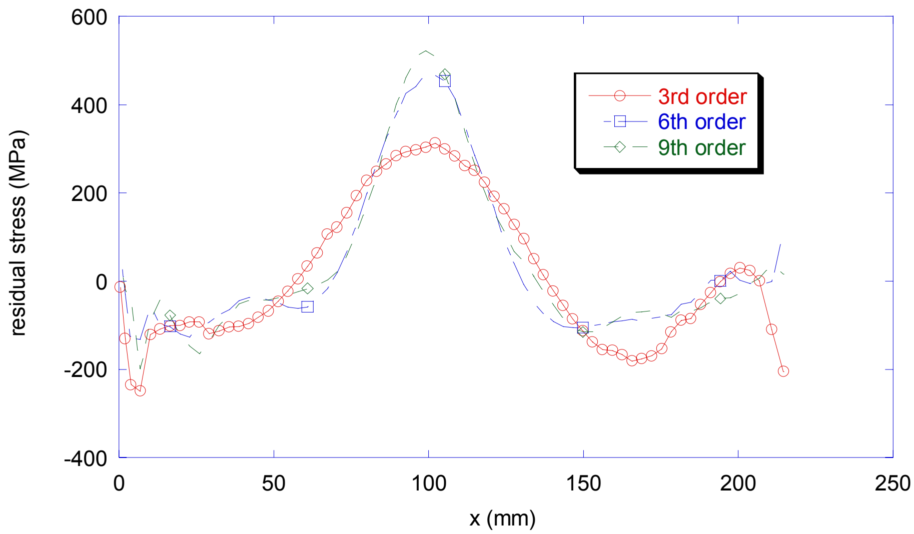


Figure 23 – Residual stress for various orders of Fourier fit, for the line y = 5.2 mm

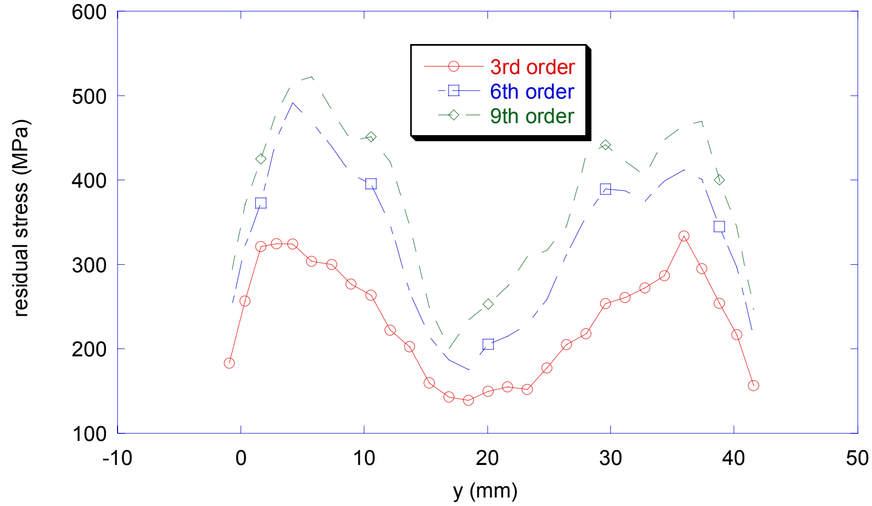


Figure 24 - Residual stress for various orders of Fourier fit, for the line x = 99.0 mm

A check on the out of plane dimension of the finite element model revealed that the residual stress was accurately estimated using the mesh shown in Figure 3. This check was performed because a small but significant level of residual stress was found on the back face of the model in the stress analysis. The length was doubled to determine if the model length had a significant effect on the estimated stress. It was found that the model length had only a small effect on the residual stress (Figure 25 and Figure 26).

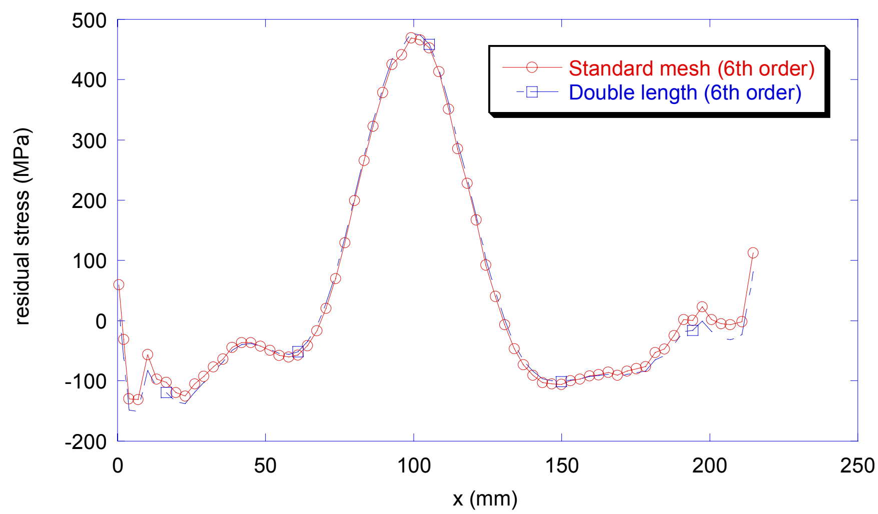


Figure 25 – Effect of doubling the length of the FE model along the line y = 5.2 mm

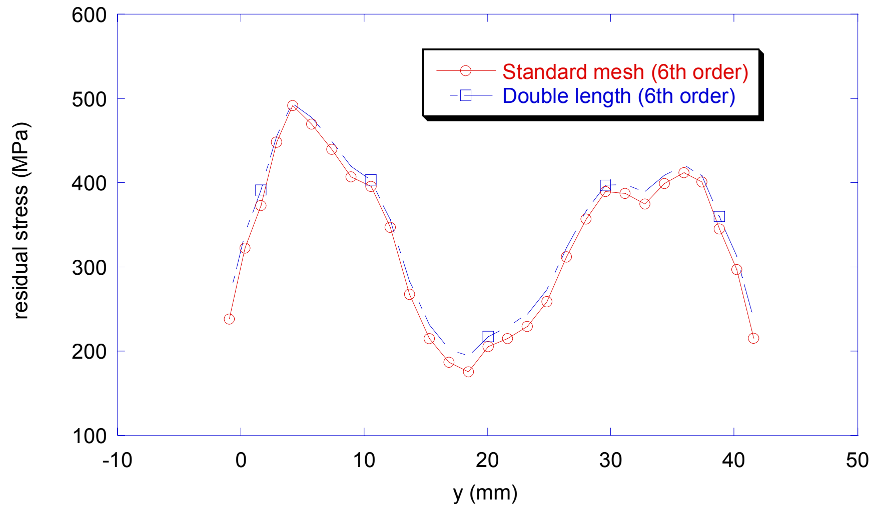


Figure 26 – Effect of doubling the length of the FE model along the line x = 99 mm

The truncation of erroneous CMM data near the surface edges required extrapolation of the surface fit when generating nodal displacements for the stress analysis, and the extrapolation method had a significant effect on the residual stress determined. We employed a plateau function, where the displacement of nodes outside the region of valid data (Figure 4) was set equal to the displacement of the nearest node within the region of valid data. Another method considered for extrapolation was to stretch the Fourier domain from the usable data range (i.e. x ~ [5 mm, 213 mm] and y ~ [2 mm, 36.5 mm]) to the entire range of the FE model (i.e., x ~ [-1 mm, 215 mm] and y ~ [-1 mm, 41.5 mm). This was accomplished by inserting the values of minimum and maximum coordinates of the FE mesh into Equation (2), in place of the minimum and maximum coordinates of the usable data range, and using the new Fourier domain coordinates with the fit parameters previously determined. The domain stretch significantly altered the residual stress throughout the weld (Figure 27). It was expected that extrapolation of displacements would only influence the residual stress near the areas of extrapolation. Since the domain stretch affected residual stress far from the edges, the method was unsuitable.

The results obtained when using the plateau to extrapolate the surface fit were significantly influenced by the removal of the planar portion of the surface fit (i.e., the use of Equation (5)). To illustrate this fact, the stress analysis was repeated using displacements determined from the plateau, but without removing the planar component of the surface fit. Because the Fourier surface fit had considerable slope in the y-direction (Figure 12 and Figure 14), the plateau resulted a slope change in the displacement field at the limits of the valid data. The slope change created a stress peak at the boundary of the region of valid data (Figure 28), which did not occur when the planar component of the surface was removed prior to the stress analysis. Since the stress peak was an artifact of data extrapolation, and because the peak was minimized when the planar component of the surface was removed from the analysis, the use of Equation (5) was a necessary step in the analysis. In addition, comparison of the stresses produced by these two analyses further demonstrates that the plateau was preferable to the domain stretch because the effect of the plateau was localized near the boundary of the valid surface data.

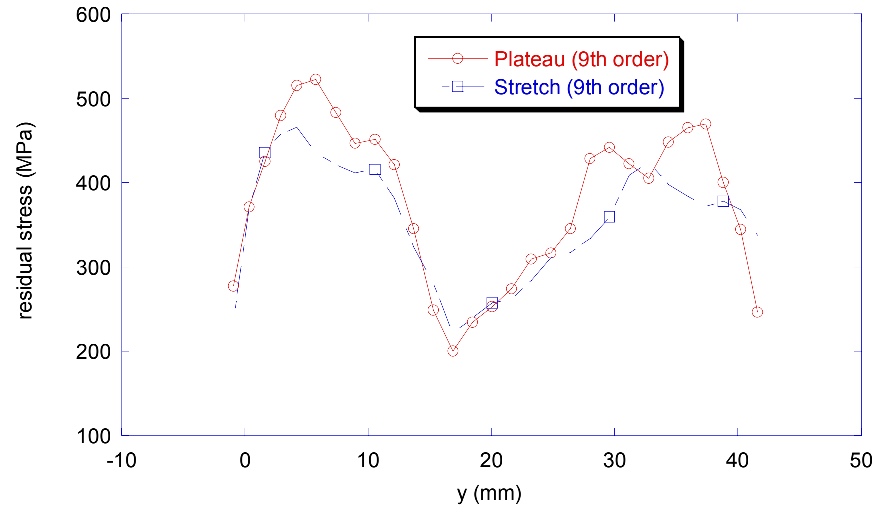


Figure 27 – Comparison of residual stress determined using a plateau or a stretch to compute nodal displacements outside the region of valid surface data, along the line x = 99 mm

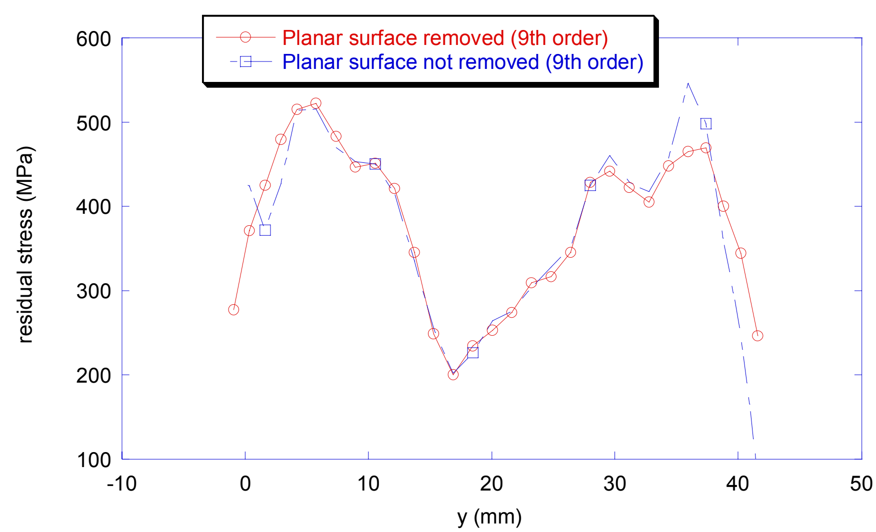


Figure 28 – Comparison of residual stress determined using a plateau with or without removal of the planar component of the surface fit, along the line x = 99 mm

Surface fitting also influenced the residual stress. Because the average surface was smooth in comparison to the original two surfaces, it was used directly to determine the residual stress for comparison with the stress found when using the fitted surface. The results of this analysis were in agreement with the results obtained from the fitted surface, but exhibited local peaks that were likely produced by either uncertainty in the CMM surface data or surface roughness due to cutting (Figure 29 and Figure 30). Because the results obtained from the fitted surface are intuitively less sensitive to point-wise uncertainties in the CMM data and to small-scale cut roughness, but otherwise produce a similar stress field, the fitted surface was beneficial.

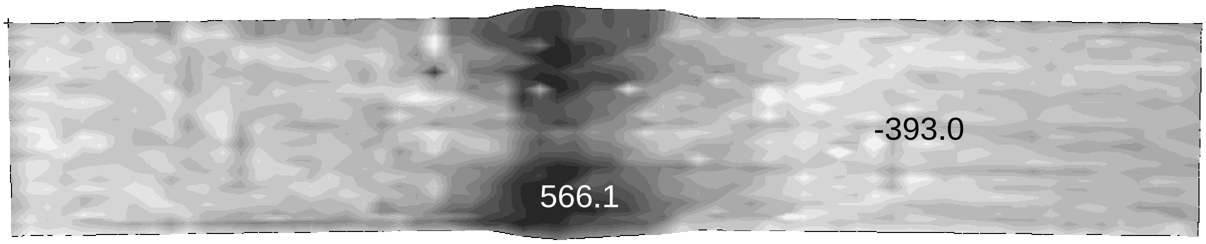


Figure 29 - z- component of residual stress for average surface without any fitting

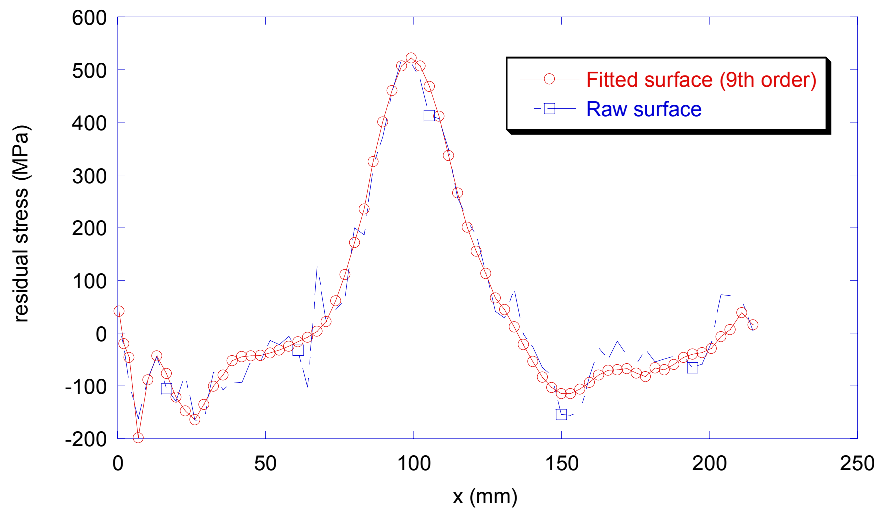
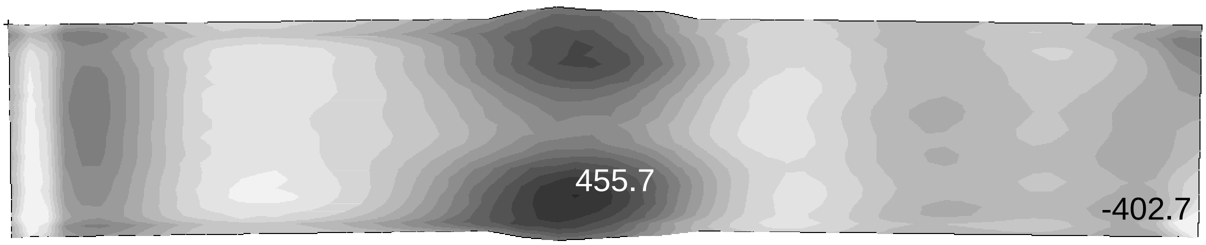
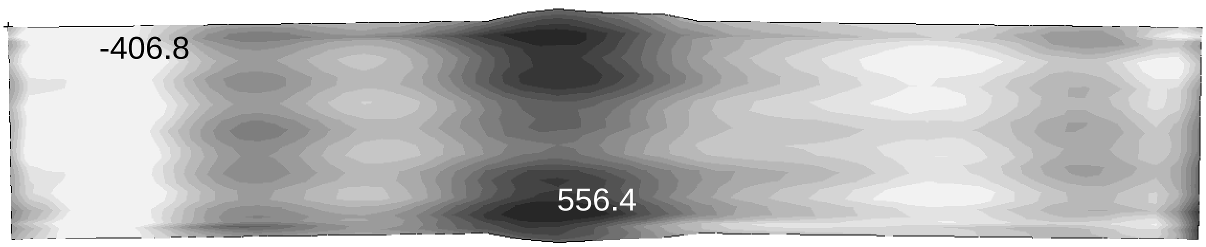


Figure 30 – Residual stress computed from raw averaged surface data and from a 9^{th} order Fourier surface fit, on the line y = 5.2 mm

The effect of surface averaging was quantified by computing residual stress from surfaces separately fitted to data from each of the two cut surfaces. Each surface produced stresses that varied significantly from stresses determined from the fit to the average surface (Figure 31 and Figure 32). The differences in the residual stress fields from these two analyses are a result of the large ridges on the individual surfaces caused by deviation from a straight cut path. Because surface averaging minimized the effects of cut-path variations, it was beneficial.



surface 1



surface 2

Figure 31 – Residual stress computed from 6th order Fourier surface fits to data from each cut surface

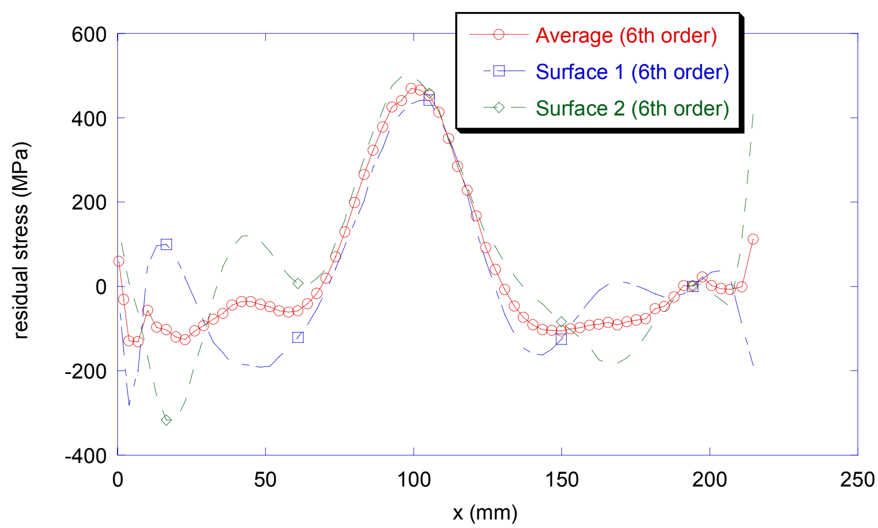


Figure 32 – Residual stress computed from 6th order Fourier surface fits to data from each cut surface and from the average surface, on the line y = 5.2 mm

CONCLUSIONS

- The weld-direction residual stress present in the welded plate had a maximum tensile magnitude of 500 MPa which occurred below the surface on the top and bottom of the weld. Compressive residual stress exists away from the weld to maintain equilibrium and is of smaller magnitude (-150 MPa).

- Ridges in the cut surfaces, presumably due to cut path variations, provided a means for verifying the alignment of the two surfaces from opposite sides of the cut.

- The averaged surface profile was markedly smoother than either of the measured surface profiles from opposite sides of the cut.

- Extrapolation of data using a plateau was found to only affect residual stress near the area of extrapolation; extrapolation by domain stretching was found to affect residual stress in the entire domain.

- The effect of plateau extrapolation was minimized by removing the planar component of surface prior to extrapolation.

- Fitting a smooth surface to the averaged surface profile had a minimal effect on the overall residual stress distribution, except where it removed localized peaks.

- Averaging of the surface profiles from opposite sides of the cut had a significant impact of the residual stress field.

APPENDICES

MATLAB code for surface fitting

```
% ADRIAN DEWALD
% 6 June 2000
% Program to fit surface with Fourier Series
% modified by mhill 6/8/00
% modified by Adrian DeWald 8/27/01

clear all;
close all;

theta2xy=-.82*pi/180.;

%load old data from Mike Prime (Convex side)
load Lba_nohead.txt
load Lbb_nohead.txt
x1=[Lbb_nohead(:,1);Lba_nohead(:,1)];
y1=[Lbb_nohead(:,3);Lba_nohead(:,3)];
z1=[Lbb_nohead(:,2);Lba_nohead(:,2)];
clear Lbb_nohead;
clear Lba_nohead;

%load new data from Mike Prime (Concave side)
load newdata.txt
x2=[newdata(:,1)];
y2=[newdata(:,2)];
z2=[newdata(:,3)];
clear newdata;

%Look at raw data
[Xiraw,Yiraw]=meshgrid(-1:.25:225,-1:.25:45);
%TempRaw1=griddata(x1,y1,z1,Xiraw,Yiraw,'cubic');
%TempRaw2=griddata(x2,y2,z2,Xiraw,Yiraw,'cubic');
%figure(1)
%subplot(2,1,1)
%mesh(Xiraw,Yiraw,TempRaw1)
%axis([-5,220,0,45,-.25,.25])
%title('Raw data for old half')
%xlabel('mm');
%ylabel('mm');
%subplot(2,1,2)
%mesh(Xiraw,Yiraw,TempRaw2)
%axis([-5,220,0,45,-.25,.25])
%title('Raw data for new half')
%xlabel('mm');
%ylabel('mm');

%translate and rotate the new data
y2=38-y2;
x2old=x2;
y2old=y2;
x2t=x2';
y2t=y2';
x2=cos(theta2xy)*x2old+sin(theta2xy)*y2old;
y2=-sin(theta2xy)*x2old+cos(theta2xy)*y2old;

%%Look at the data sets after rotation
%figure(2)
%subplot(2,1,1)
%plot(x1,y1)
%axis([-50,250,-10,40])
%title('Plot of profile view of old data after rotation')
%xlabel('mm');
%ylabel('mm');
%subplot(2,1,2)
%plot(x2,y2)
%axis([-50,250,-10,40])
%title('Plot of porfile view of new data after rotation')
%xlabel('mm');
%ylabel('mm');

%weld has dimensions 214mm by 38mm but take points out of a
rectangle
%and don't pick up any NaN's, and don't pick up any points
were probe fell off surface
[Xi,Yi]=meshgrid(5:.5:213,2:.5:36.5);

% Use DeLaunay Triangulation to grid data
Temp1=griddata(x1,y1,z1,Xi,Yi,'cubic');
Temp2=griddata(x2,y2,-z2,Xi,Yi,'cubic');

% Test to see how data looks
figure(3)
surf(Temp1)
title('griddata for old half')
xlabel('mm');
ylabel('mm');
figure(4)
```

```
surf(Temp2)
title('griddata for new half')
xlabel('mm');
ylabel('mm');

Ti=size(Temp1,2);
Tmin=min(Xi(1,:));
Tmax=max(Xi(1,:));
T=Xi(1,:)';
Si=size(Temp1,1);
Smin=min(Yi(:,1));
Smax=max(Yi(:,1));
S=Yi(:,1);

% Average the data
TempAvg=(Temp1+Temp2)/2;
Tempdiff=Temp1-Temp2;
figure(6)
subplot(2,1,1);
mesh(T,S,Temp1);
view(38,24);
title(['average of the two surfaces']);
hold on
mesh(T,S,Temp2);
surf(T,S,TempAvg);
hold off
axis([Tmin Tmax Smin Smax -.05 .2])
xlabel('mm');
ylabel('mm');
subplot(2,1,2);
surf(T,S,Tempdiff);
view(38,24);
caxis([-.1 .1])
colorbar
title(['difference between two surfaces']);
xlabel('mm');
ylabel('mm');

%% Show where useable data is coming from
%figure(2)
%subplot(2,1,1)
%hold on
%plot(Xi,Yi,'r')
%axis([-50,250,-10,40])
%subplot(2,1,2)
%hold on
%plot(Xi,Yi,'r')
%axis([-50,250,-10,40])
%hold off

Rows=Ti*Si;
U_hat=zeros(Rows,1);
l=0;
h=1;

Tplus=zeros(Rows,1);
Splus=zeros(Rows,1);
count=1;
for l=1:Si;
   h=1;
   for h=1:Ti;
      U_hat(count)=TempAvg(l,h);
      Tplus(count)=T(h,1);
      Splus(count)=S(l,1);
      count=count+1;
  end
end

% Transform Data to Fourier Domain
% Include lack of syppetry in T direction (change 2*pi*...
to just pi*...)
Tplust=pi*(Tplus-Tmin)/(Tmax-Tmin);
%  -- Include lack of symmetry in S direction
Splust=pi*(Splus-Smin)/(Smax-Smin);

i=1;
m=input ('What order of fit (n)? ');
Columns=1+2*m*(m+1);
%For sines and cosines
C=zeros(Rows,Columns);
l=0;
h=0;
C(:,1)=ones(Rows,1);

index=2;
count=1;
while (index < Columns);
   for l=0:count;
      h=count-l;
      if(l == 0)
        C(:,index)=cos(Tplust*h);
        index = index + 1;
        C(:,index)=sin(Tplust*h);
        index = index + 1;
      elseif(h == 0)
```

```
        C(:,index)=cos(Splust*l);
        index = index + 1;
        C(:,index)=sin(Splust*l);
        index = index + 1;
      else
        C(:,index)=cos(Tplust*h).*cos(Splust*l);
         index = index + 1;
        C(:,index)=sin(Tplust*h).*cos(Splust*l);
         index = index + 1;
        C(:,index)=cos(Tplust*h).*sin(Splust*l);
         index = index + 1;
        C(:,index)=sin(Tplust*h).*sin(Splust*l);
         index = index + 1;
      end
   end
   count=count+1;
end

A=(C'*C)\C'*U_hat;
At=rot90(A,-1);
f=C*A;
Clin=zeros(size(C(:,1:4)));
Clin(:,1)=ones(Rows,1);
Clin(:,2)=Tplust;
Clin(:,3)=Splust;
Clin(:,4)=Tplust.*Splust;
Alin=(Clin'*Clin)\Clin'*f;
flin=Clin*Alin;
frot=f-flin;
ZI=zeros(Si,Ti);
ZIlin=zeros(Si,Ti);
ZIrot=zeros(Si,Ti);
k=1;
l=1;
count=1;
for k=1:Si;
   for l=1:Ti;
      ZI(k,l)=f(count);
      ZIlin(k,l)=flin(count);
      ZIrot(k,l)=frot(count);
      count=count+1;
  end
end
count=1;
figure(5)
subplot(2,1,1);
mesh(T,S,TempAvg);
hold on
surf(T,S,ZI);
view(38,24);
xlabel('mm');
ylabel('mm');
hold off
axis([Tmin Tmax Smin Smax -.05 .25])
errors=f-U_hat;
rms_error=norm(errors)/sqrt(size(errors,1)-Columns)
title(['Two halves averaged Fourier fit for order
',num2str(m),' with ',num2str(Columns),' terms. RMS error
is ',num2str(rms_error)])
subplot(2,1,2);
surf(T,S,reshape(errors',size(TempAvg'))')
view(38,24);
axis([Tmin Tmax Smin Smax -.02 .025])
caxis([-0.01 0.01])
colorbar
title(['Error in the Fit']);
xlabel('mm');
ylabel('mm');

% show the plane fit to the surface
figure(8)
mesh(T,S,ZI);
hold on
surf(T,S,ZIlin);
view(38,24);
xlabel('mm');
ylabel('mm');
hold off
axis([Tmin Tmax Smin Smax -.05 .25])
title(['Two halves averaged Fourier fit for order
',num2str(m),' with ',num2str(Columns),' terms. RMS error
is ',num2str(rms_error)])

% show the surface before and after rotation
figure(9)
mesh(T,S,ZI);
hold on
surf(T,S,ZIrot);
view(38,24);
xlabel('mm');
ylabel('mm');
hold off
axis([Tmin Tmax Smin Smax -.05 .25])

% make output file for A's
```

```
Asave=[A;Alin];
save Awlin.txt Asave -ASCII

% plot the edges of each half to check fit
figure(7)
subplot(2,2,1);
plot(Temp1(1,:),'b');
hold on
plot(Temp2(1,:),'r');
plot(TempAvg(1,:),'g');
Title(['Edge where y = ',num2str(Yi(1,1))]);
legend('old','new','avg');
axis([0,450, -.1, .2]);
hold off
xlabel('mm');
ylabel('mm');
subplot(2,2,2);
plot(Temp1(:,1),'b');
hold on
plot(Temp2(:,1),'r');
plot(TempAvg(:,1),'g');
Title(['Edge where x = ',num2str(Xi(1,1))]);
legend('old','new','avg');
axis([0,80, -.03, .01]);
xlabel('mm');
ylabel('mm');
hold off
subplot(2,2,3);
plot(Temp1(70,:),'b');
hold on
plot(Temp2(70,:),'r');
plot(TempAvg(70,:),'g');
Title(['Edge where y = ',num2str(Yi(70,1))]);
legend('old','new','avg');
axis([0,450, -.1, .2]);
xlabel('mm');
ylabel('mm');
hold off
subplot(2,2,4);
plot(Temp1(:,417),'b');
hold on
plot(Temp2(:,417),'r');
plot(TempAvg(:,417),'g');
Title(['Edge where x = ',num2str(Xi(1,417))]);
legend('old','new','avg');
axis([0,80, -.15, .05]);
xlabel('mm');
ylabel('mm');
hold off
```

MATLAB code for obtaining nodal displacements

```
% ADRIAN DEWALD
% 6 June 2000
% Program to fit surface with Fourier Series
% modified by mhill 6/8/00
% modified to go with new fsurf dimensions 8/01

clear all

load blo_ref_z0nodes
node=blo_ref_z0nodes(:,1);
x=25.4*blo_ref_z0nodes(:,2)-0.5;
y=39.14-0.9+25.4*blo_ref_z0nodes(:,3);
Rows = length(x);

%filter the data
count=1;
for i=1:length(x)
  if(x(count) < 5)
    x(count)=5;
  elseif(x(count) > 213)
    x(count)=213;
  end
  count=count+1;
end
count=1;
for i=1:length(y)
  if(y(count) < 2)
    y(count)=2;
  elseif(y(count) > 36.5)
    y(count)=36.5;
  end
  count=count+1;
end

% Transform Data to Fourier Domain
xt=pi*(x-5)/(213-5);
%  -- Include lack of symmetry in S direction
yt=pi*(y-2)/(36.5-2);

i=1;
m=input ('What order of fit (n)? ');

Columns=1+2*m*(m+1);
l=0;
h=0;
C(:,1)=ones(Rows,1);
index=2;
count=1;
while (index < Columns);
   for l=0:count;
      h=count-l;
      if(l == 0)
        C(:,index)=cos(xt*h);
        index = index + 1;
        C(:,index)=sin(xt*h);
        index = index + 1;
      elseif(h == 0)
        C(:,index)=cos(yt*l);
        index = index + 1;
        C(:,index)=sin(yt*l);
        index = index + 1;
      else
        C(:,index)=cos(xt*h).*cos(yt*l);
     index = index + 1;
        C(:,index)=sin(xt*h).*cos(yt*l);
     index = index + 1;
        C(:,index)=cos(xt*h).*sin(yt*l);
     index = index + 1;
        C(:,index)=sin(xt*h).*sin(yt*l);
     index = index + 1;
      end
   end
   count=count+1;
end
load Awlin.txt

% take out the coefficients for x, y, and xy
Clin=zeros(Rows,4);
Clin(:,1)=ones(Rows,1);
Clin(:,2)=xt;
Clin(:,3)=yt;
Clin(:,4)=xt.*yt;
f=C*Awlin(1:Columns)-Clin*Awlin(Columns+1:length(Awlin));

%convert to English units
displ=f/25.4;
save displ_Temp.txt displ -ASCII
```

Sample ABAQUS input file for stress analysis

```
*HEADING, SPARSE
ABAQUS job created on 21-Jul-00 at 17:46:23
**
*NODE
     1,     3.65,     0.103269
     2,     3.65,     0.008041
     3,      3.9,     0.131263
     4,      3.9,     0.021893
     5,     4.15,      0.10888
     6,     4.15,     0.011396
     7,      4.4,     0.100833
     8,      4.4,     0.008776
     9,     4.65,     0.097005
    10,     4.65,      0.00289
     .        .          .
     .        .          .
nonessential information left out here
     .        .          .
```

```
   .          .          .
32001,      8.309,   -0.0988871,        -5.
32002,    8.47029,    -0.100857,        -5.
32003,     8.3093,   -0.0651383,        -5.
32004,    8.47094,   -0.0671124,        -5.
32005,    8.30953,   -0.0314273,        -5.
32006,    8.47144,   -0.0333676,        -5.
32007,    8.30975,   0.00228374,        -5.
32008,    8.47194,  0.000377178,        -5.
32009,    8.30001,     -1.45006,        -5.
32010,    8.30024,     -1.41631,        -5.
32011,    8.45038,      -1.4519,        -5.
32012,    8.45088,     -1.41815,        -5.
32013,    8.29956,     -1.51801,        -5.
32014,    8.29979,     -1.48403,        -5.
32015,    8.44937,     -1.51983,        -5.
32016,    8.44987,     -1.48586,        -5.
**
**
*ELEMENT, TYPE=C3D8I, ELSET=PID0
    1,   527,   530,   526,    33,  2002,  2005,
  2004,  2003
    2,   530,   528,    22,   526,  2005,  2007,
  2006,  2004
    3,    32,   529,   530,   527,  2008,  2009,
  2005,  2002
    4,   529,    21,   528,   530,  2009,  2010,
  2007,  2005
    5,   532,   534,   531,    34,  2011,  2014,
  2013,  2012
    6,   534,   533,    23,   531,  2014,  2016,
  2015,  2013
    7,    33,   526,   534,   532,  2003,  2004,
  2014,  2011
    8,   526,    22,   533,   534,  2004,  2006,
  2016,  2014
    9,   536,   538,   535,    35,  2017,  2020,
  2019,  2018
   10,   538,   537,    24,   535,  2020,  2022,
  2021,  2019
      .      .
      .      .
nonessential information left out here
      .      .
      .      .
 28550, 30004, 30005, 30003, 30002, 32005, 32006,
 32004, 32003
 28551, 29807, 30006, 30004, 29804, 31808, 32007,
 32005, 31805
 28552, 30006, 30007, 30005, 30004, 32007, 32008,
 32006, 32005
 28553, 29859, 30009, 30008, 29858, 31860, 32010,
 32009, 31859
 28554, 30009, 30011, 30010, 30008, 32010, 32012,
 32011, 32009
 28555, 29265, 29996, 30009, 29859, 31266, 31997,
 32010, 31860
 28556, 29996, 29998, 30011, 30009, 31997, 31999,
 32012, 32010
 28557, 29911, 30013, 30012, 29910, 31912, 32014,
 32013, 31911
 28558, 30013, 30015, 30014, 30012, 32014, 32016,
 32015, 32013
 28559, 29858, 30008, 30013, 29911, 31859, 32009,
 32014, 31912
 28560, 30008, 30010, 30015, 30013, 32009, 32011,
 32016, 32014
**
**
*BOUNDARY, OP=NEW, FIXED
   315,  1,,   0.0
   315,  2,,   0.0
   525,  2,,   0.0
*PREPRINT, ECHO=NO, MODEL=NO, HISTORY=NO
*SOLID SECTION, ELSET=PID0, MATERIAL=ZIP
*MATERIAL, NAME=ZIP
*ELASTIC
 30.00E06, .292
*STEP,AMPLITUDE=STEP,PERTURBATION
*STATIC
1.0,1.0,1.0,1.0
*BOUNDARY, OP=MOD
1, 3,, 3.93E-03
2, 3,, 3.93E-03
3, 3,, 4.04E-03
4, 3,, 4.04E-03
5, 3,, 3.75E-03
6, 3,, 3.75E-03
7, 3,, 3.17E-03
8, 3,, 3.17E-03
9, 3,, 2.47E-03
10, 3,, 2.47E-03
    .      .
    .      .
nonessential information left out here
    .      .
    .      .
1992, 3,, -1.25E-03
1993, 3,, -1.09E-03
1994, 3,, -1.28E-03
1995, 3,, -1.28E-03
1996, 3,, -5.93E-04
1997, 3,, -4.28E-04
1998, 3,, -6.01E-04
1999, 3,, -5.93E-04
2000, 3,, -4.19E-04
2001, 3,, -5.93E-04
**
*EL PRINT, POSITION=AVERAGED AT NODES, FREQUENCY=0
*NODE PRINT, GLOBAL=YES, TOTALS=YES, FREQUENCY=0
***NODE PRINT, GLOBAL=YES, TOTALS=YES, NSET=MIDPL
** RF
*NODE FILE
 U
*EL FILE, POSITION=INTEGRATION POINTS
 S,E
*EL FILE, POSITION=AVERAGED AT NODES
 S,E
*END STEP
```